Discussion of the Observations of the Satellite of Neptune made at the Royal Observatory, Greenwich, in the years 1902–3–4. By F. W. Dyson, M.A., F.R.S., and D. J. R. Edney.

The observations discussed in this paper are published in the *Monthly Notices*, vols. lxii., lxiii., and lxiv., and are obtained from photographs with the 26-inch refractor taken with the aid of an occulting shutter. Specimen photographs are given in *Monthly Notices*, vol. lxii. p. 623, with an account of the occulting shutter and method and details of measurement. The published measures are compared with tabular places obtained from the *Connaissance des Temps* based on Mr. Hermann Struve's elements.

The notation and formulæ employed in obtaining corrections to the elements are taken from Mr. Struve's discussion in the *Mémoires de l'Académie Impériale des Sciences de St. Pétersbourg*, vii^{e} série, tome xlii., No. 4. As Mr. Struve has discussed the observations prior to 1892, and as his elements are employed in the *Connaissance des Temps*, it is desirable and convenient to keep closely to his form. The only difference in notation is that, following the *Connaissance*, +U is written where Mr. Struve has −U.

Assuming the orbit of the satellite to be circular, its distance s and position-angle p are given by the formulæ

$$s \sin (p-\mathrm{P}) = r \sin (u+\mathrm{U})$$

$$s \cos (p-\mathrm{P}) = r \sin \mathrm{B} \cos (u+\mathrm{U})$$

$$r = a \frac{(\rho)}{\rho'}\,;\ \rho' = \rho(1 + a \cos \mathrm{B} \cos (u+\mathrm{U}) \sin 1'')$$

where a is the elongation at mean distance (ρ) of *Neptune* ;

ρ is the geocentric distance of *Neptune* ;

u is the longitude of the satellite measured from the node of its orbit on the Earth's equator ;

$180° - \mathrm{U}$ and B are the planetocentric longitude and latitude of the Earth with reference to the orbit of the satellite ;

and P is the position-angle of the pole of the orbit of the satellite ;

U, B, P are readily expressible in terms of N, I, the longitude of the node and inclination, and α, δ the right ascension and declination of the planet. Their values along with $a \frac{(\rho)}{\rho}$ and u are tabulated in the *Connaissance des Temps*.

The formulæ for correcting the elements are given by Mr. H. Struve in the form

$$\begin{aligned} s \sin dp = {} & r \sin \tau \,.\, \sin du \\ & + (r \sin \tau \cos \mathrm{I} + r \cos \tau \cos u \sin \mathrm{I}) \,.\, \sin d\mathrm{N} \\ & - r \cos \tau \sin u \,.\, \sin d\mathrm{I} \\ & - r \sin \tau \cos u \,.\, 2e \sin \mathrm{Q} \\ & + r \sin \tau \sin u \,.\, 2e \cos \mathrm{Q}\,; \end{aligned}$$

$$\begin{aligned} ds = {} & r \cos \sigma \cos \tau \,.\, \sin du \\ & + r \cos \sigma \sin p \cos \delta \,.\, \sin d\mathrm{N} \\ & + r \cos \sigma \sin \tau \sin u \,.\, \sin d\mathrm{I} \\ & - \left(r \cos \sigma \cos \tau \cos u + \frac{s}{2} \sin u \right) .\, 2e \sin \mathrm{Q} \\ & + \left(r \cos \sigma \cos \tau \sin u - \frac{s}{2} \cos u \right) .\, 2e \cos \mathrm{Q} \\ & + s \,.\, \frac{da}{a} \end{aligned}$$

where e is the eccentricity, and Q the longitude of periastron measured from the node of the satellite's orbit on the Earth's equator.

And the auxiliaries τ and σ are defined by the equations:

$$\sin \tau = \frac{r}{s} \sin \mathrm{B}$$

$$\cos \tau = \frac{r}{s} \cos \mathrm{B} \sin (u + \mathrm{U})$$

$$\cos \sigma = \cos \mathrm{B} \cos (u + \mathrm{U}).$$

The adopted value of N and I in the above formulæ are

$$\mathrm{N} = 185^\circ\!\cdot\!15 + 0^\circ\!\cdot\!148\ (t - 1890)$$
$$\mathrm{I} = 119\cdot35 - 0\cdot165\ (t - 1890)$$

The equations of condition obtained by applying the above formulæ to the results of the Greenwich photographs are given below for the three oppositions considered. The residuals are given in the last column. Taking $e=0$ and adopting the mean values for du, $d\mathrm{N}$, $d\mathrm{I}$, and $\frac{da}{a}$, the probable error for weight 1, *i.e.* for a result derived from one photograph, is $\pm 0''\!\cdot\!14$ for both the distances and position-angles.

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Equations of Condition.

Position-angle.

Date.	Weight.	sin *du*.	sin *d*N.	sin *d*I.	2*e* sin Q.	2*e* cos Q.		*s* sin *dp*.	Residual.
1902. Jan. 6	2	−14″·1	+1″·0	−6″·9	− 9″·5	−10″·5	=	+0″·18	−0″·05
14	2	−15·9	+6·7	+5·6	− 2·2	+15·8	=	+ ·21	+ ·01
24	2	−12·5	−2·8	−5·9	−10·7	− 6·5	=	+ ·35	+ ·11
28	2	−11·8	−2·7	+7·9	+ 8·9	− 7·7	=	+ ·29	+ ·10
29	2	−16·8	+7·8	+0·3	− 2·6	−16·6	=	+ ·36	+ ·14
30	1	−11·9	−4·2	−4·9	−10·8	− 4·9	=	+ ·39	+ ·15
31	1	−11·5	−3·7	+6·9	− 9·5	+ 6·4	=	·00	− ·21
Feb. 10	2	−16·4	+6·6	−3·0	− 5·4	−15·5	=	+ ·21	− ·02
11	4	−11·2	−5·6	−2·7	−10·9	− 2·5	=	+ ·20	− ·03
12	1	−12·2	−1·1	+8·6	− 8·1	+ 9·1	=	+ ·14	− ·05
13	1	−16·3	+6·3	−3·4	+ 5·7	+15·2	=	+ ·36	+ ·13
15	2	−12·6	+0·1	+8·8	+ 7·4	−10·2	=	+ ·13	− ·06
16	2	−16·1	+5·9	−4·1	− 6·3	−14·8	=	+ ·44	+ ·20
28	3	−14·0	+1·5	−6·6	− 9·2	−10·7	=	·00	− ·23
Mar. 1	1	−10·6	−6·2	+2·5	−10·4	+ 2·1	=	+ ·53	+ ·32
3	5	−13·7	+0·7	−6·6	+ 9·5	+ 9·9	=	+ ·17	− ·06
17	1	−16·3	+7·6	+1·8	− 1·4	−16·3	=	+ ·41	+ ·20
19	3	−11·0	−4·0	+6·7	− 9·2	+ 6·0	=	+ ·13	− ·07
21	3	−11·5	−4·2	−4·6	+10·6	+ 4·5	=	+ ·24	+ ·01
22	1	−11·4	−2·8	+7·6	+ 8·7	− 7·3	=	+ ·15	− ·04
25	3	−11·4	−2·7	+7·7	− 8·6	+ 7·4	=	+ ·27	+ ·08
27	4	−11·1	−5·2	−3·3	+10·6	+ 3·0	=	+ ·20	− ·03
Apr. 6	1	−12·4	+0·4	+8·6	− 7·1	+10·2	=	+ ·08	− ·10
10	2	−14·8	+3·8	−5·6	− 7·7	−12·7	=	+ ·22	·00

Equations of Condition.

Distance.

Date.	Weight.	sin *du*.	sin *d*N.	sin *d*I.	2*e* sin Q.	2*e* cos Q.	$\frac{da}{a}$.		*s* sin *dp*.	Residual.
1902. Jan. 6	2	+6″·0	−8″·1	− 6″·8	−0″·7	+8″·7	+12″·9	=	+0″·10	+″·06
14	2	−4·1	+3·2	−11·6	+5·0	+4·9	+11·4	=	+ ·23	+·10
24	2	+5·7	−7·3	− 3·3	+1·1	+9·1	+14·5	=	+ ·09	+·04
28	1	−4·8	+5·4	− 3·1	−1·2	−8·8	+15·4	=	+ ·12	−·01
29	2	−0·2	−1·7	−12·7	−5·3	+0·6	+10·7	=	+ ·08	−·02
30	1	+5·0	−6·3	− 2·1	+1·5	+8·9	+15·2	=	− ·06	−·11
31	1	−4·2	+4·8	− 2·2	+1·0	+9·0	+15·9	=	− ·21	−·32
Feb. 10	2	+2·4	−4·7	−11·8	−4·4	+4·1	+10·9	=	− ·08	−·15

Date.	Weight.	sin *du*.	sin *d*N.	sin *d*I.	2*e* sin Q.	2*e* cos Q.	$\frac{da}{a}$.	*s* sin *dp*.	Residual.
1902.		″	″	″	″	″	″	″	″
Feb. 11	4	+3·7	−4·5	− 0·7	+1·8	+8·6	+16·0 =	+ ·03	−·02
12	1	−5·5	+5·9	− 4·4	+1·8	+9·0	+14·6 =	− ·11	−·25
13	1	+2·8	−5·1	−11·5	+4·2	−4·5	+10·9 =	+ ·28	+·21
15	2	−5·8	+6·2	− 5·5	−2·3	−8·9	+14·1 =	+ ·22	+·09
16	2	+3·3	−5·7	−11·1	−3·8	+5·2	+11·1 =	+ ·07	·00
28	3	+5·7	−8·1	− 7·1	−1·0	+8·4	+12·4 =	+ ·17	+·12
Mar. 1	1	−0·2	+0·2	0·0	+1·4	+8·3	+16·6 =	+ ·15	+·06
3	5	+5·9	−8·1	− 6·3	+0·5	−8·6	+12·7 =	+ ·03	−·01
17	1	−1·4	−0·2	−12·5	−5·4	−0·9	+10·5 =	− ·17	−·28
19	3	−3·7	+4·2	− 1·8	+1·2	+8·5	+15·6 =	+ ·21	+·10
21	3	+4·9	−6·2	− 1·9	−1·6	−8·8	+14·8 =	+ ·10	+·05
22	1	−4·6	+5·1	− 2·8	−1·4	−8·7	+15·1 =	+ ·23	+·10
25	3	−4·6	+5·1	− 2·9	+1·4	+8·7	+15·0 =	+ ·06	−·07
27	4	+4·0	−5·0	− 1·0	−1·8	−8·5	+15·4 =	− ·06	−·11
Apr. 6	1	−5·7	+6·1	− 5·6	+2·3	+8·6	+13·6 =	+ ·24	+·12
10	2	+4·7	−7·1	− 9·1	−2·4	+6·9	+11·3 =	+ ·22	+·17

Equations of Condition.

Position-angle.

Date.	Weight.	sin *du*.	sin *d*N.	sin dI.	2*e* sin Q.	2*e* cos Q.	*s* sin *dp*.	Residual.
1902.		″	″	″	″	″	″	″
Nov. 12	4	−12·5	−1·3	+7·9	+ 8·9	− 8·8 =	−0·15	−·27
13	2	−16·5	+6·9	−3·0	− 3·8	−16·1 =	− ·19	−·35
17	1	−11·2	−5·8	−2·7	+10·9	+ 2·4 =	+ ·49	+·31
28	2	−14·7	+2·6	−6·9	+ 8·1	+12·5 =	− ·04	−·23
Dec. 29	2	−11·6	−4·1	+6·4	+ 9·9	− 6·0 =	− ·13	−·26
31	2	−11·6	−5·1	−4·1	−10·9	− 3·9 =	+ ·06	−·13
1903. Jan. 1	1	−12·7	−0·9	+8·3	− 8·6	+ 9·4 =	− ·03	−·14
3	2	−11·5	−5·2	−3·9	+10·9	+ 3·7 =	+ ·64	+·45
15	3	−11·0	−6·4	−0·3	+11·0	+ 0·3 =	+ ·28	+·11
23	3	−14·5	+2·0	−6·9	− 8·7	−11·6 =	+ ·50	+·30
25	1	−14·6	+3·9	+7·7	− 5·5	+13·6 =	− ·28	−·38
28	2	−14·7	+4·1	+7·6	+ 5·3	−13·7 =	+ ·29	+·19
Feb. 1	1	−13·2	−0·8	−6·9	+ 9·9	+ 8·8 =	+ ·77	+·58
2	1	−10·9	−6·0	+3·4	+10·5	− 2·9 =	+ ·22	+·07
6	1	−16·1	+6·9	+4·6	− 2·0	+16·0 =	+ ·61	+·49
10	1	−12·3	−3·1	−5·9	−10·5	− 6·4 =	+ ·07	−·12
16	2	−11·4	−5·2	−3·7	−10·8	− 3·5 =	+ ·19	·00
17	3	−11·5	−3·6	+6·8	− 9·5	+ 6·5 =	− ·05	−·17

Date.	Weight.	sin du.	sin dN.	sin dI.	2e sin Q.	2e cos Q.		s sin dp.	Resi-dual.
1902. Feb. 18	3	−16″·7	+7″·4	−1″·3	+ 3″·2	+16″·4	=	+ ″·15	″·00
23	1	−11·8	−2·6	+7·6	− 9·0	+ 7·6	=	+ ·27	+·15
26	1	−12·2	−1·4	+8·1	+ 8·5	− 8·8	=	+ ·21	+·10
28	3	−10·9	−5·9	−2·0	−10·8	− 1·7	=	+ ·29	+·11
Mar. 3	3	−10·9	−6·0	−1·6	+10·8	+ 1·4	=	− ·03	−·21
6	3	−10·7	−6·3	−0·3	−10·7	− 0·3	=	+ ·22	+·06
11	2	−14·6	+2·9	−6·4	− 8·1	−12·1	=	·00	−·18
13	1	−13·9	+3·1	+8·0	− 5·9	+12·7	=	+ ·12	+·02
15	2	−10·6	−6·0	+2·0	+10·5	− 1·6	=	+ ·23	+·08
16	2	−14·9	+5·2	+6·7	+ 4·0	−14·4	=	·00	−·10
21	2	−10·7	−5·8	+3·5	+10·2	− 3·0	=	− ·11	−·25
26	1	−12·4	−2·0	−6·3	+10·0	+ 7·3	=	+ ·36	+·18
Apr. 14	1	−11·8	−3·7	+7·7	+ 8·4	− 8·2	=	+ ·47	+·34
16	1	−10·8	−5·5	−2·6	−10·5	− 2·4	=	+ ·51	+·33
17	1	−12·1	−0·7	+8·0	− 8·0	+ 9·1	=	− ·04	−·14
24	1	−14·8	+4·0	−5·7	+ 7·0	+13·0	=	+ ·18	+·01
27	1	−14·3	+3·1	−6·2	− 7·6	−12·1	=	+ ·22	+·04

Equations of Condition.

Distance.

Date.	Weight.	sin du.	sin dN.	sin dI.	2e sin Q.	2e cos Q.	$\frac{da}{a}$.		s sin dp.	Resi-dual.
1902. Nov. 12	4	−5″·6	+6″·5	− 4″·4	−1″·2	−9″·1	+14″·6	=	+0″·10	−″·02
13	2	+2·3	−3·6	−12·1	−4·9	+3·5	+11·1	=	+ ·18	+·07
17	1	+2·8	−3·4	− 0·5	−0·9	−8·6	+16·4	=	− ·31	−·39
28	2	+5·5	−7·3	− 8·3	+2·2	−8·1	+12·6	=	− ·15	−·25
Dec. 29	2	−4·1	+4·8	− 2·0	−0·7	−8·9	+16·0	=	+ ·28	+·16
31	2	+4·0	−5·0	− 1·3	+1·1	+8·9	+16·0	=	+ ·19	+·11
1903. Jan. 1	1	−5·7	+6·5	− 4·8	+1·5	+9·1	+14·6	=	+ ·02	−·11
3	2	+3·9	−4·9	− 1·2	−1·2	−8·9	+16·1	=	+ ·19	+·11
15	3	+1·3	−1·6	0·0	−1·1	−8·4	+16·8	=	·00	−·08
23	3	+5·7	−7·7	− 7·7	−1·7	+8·4	+12·7	=	·00	−·09
25	1	−5·6	+5·8	− 9·1	+3·7	+7·5	+12·5	=	− ·19	−·33
28	2	−5·5	+5·7	− 9·3	−3·8	−7·4	+12·4	=	+ ·38	+·24
Feb. 1	1	+5·9	−7·7	− 5·0	+0·2	−9·1	+13·8	=	− ·13	−·21
2	1	−1·5	+1·7	− 0·3	−0·8	−8·4	+16·7	=	+ ·01	−·09
6	1	−3·4	+2·8	−11·9	+5·2	+4·1	+11·3	=	− ·11	−·25
10	1	+5·4	−6·9	− 3·0	+0·8	+9·1	+14·7	=	− ·49	−·58
16	2	+3·9	−4·9	− 1·1	+1·3	+8·7	+15·8	=	+ ·23	+·15

Date.	Weight.	sin du.	sin dN.	sin dI.	$2e$ sin Q.	$2e$ cos Q.	$\frac{da}{a}$.	s sin dp.	Residual.
1902.		″	″	″	″	″	″	″	″
Feb. 17	3	−4·2	+4·9	− 2·3	+0·9	+8·8	+15·6 =	+ ·20	+·08
18	3	+1·0	−2·6	−12·5	+5·1	−2·0	+10·8 =	− ·01	−·14
23	1	−4·9	+5·6	− 3·2	+1·2	+8·9	+15·1 =	− ·02	−·14
26	1	−5·4	+6·1	− 4·2	−1·5	−9·0	+14·6 =	+ ·16	+·04
28	3	+2·7	−3·3	− 0·4	+1·4	+8·4	+16·2 =	+ ·30	+·22
Mar. 3	3	+2·4	−3·0	− 0·3	−1·4	−8·4	+16·2 =	+ ·10	+·02
6	3	+1·5	−1·8	0·0	+1·3	+8·3	+16·4 =	+ ·19	+·11
11	2	+5·3	−7·4	− 8·3	−2·1	+7·7	+12·0 =	+ ·27	+·17
13	1	−5·7	+6·0	− 8·2	+3·3	+7·8	+12·6 =	+ ·21	+·07
15	2	−0·3	+0·3	0·0	−1·0	−8·2	+16·4 =	+ ·18	+·09
16	2	−4·9	+4·7	−10·1	−4·3	−6·3	+11·7 =	− ·15	−·30
21	2	−1·5	+1·8	− 0·4	−0·9	−8·2	+16·3 =	+ ·56	+·46
26	1	+5·6	−7·3	− 3·9	−0·4	−8·9	+13·9 =	+ ·16	+·07
Apr. 14	1	−5·1	+5·8	− 3·8	−1·3	−8·7	+14·3 =	− ·04	−·16
16	1	+3·1	−3·8	− 0·6	+1·3	+8·3	+15·6 =	− ·02	−·10
17	1	−5·4	+6·1	− 4·7	+1·7	+8·7	+13·9 =	− ·15	−·27
24	1	+4·6	−6·5	− 9·2	+2·8	−6·7	+11·3 =	− ·01	−·11
27	1	+5·0	−7·0	− 8·4	−2·3	+7·4	+11·6 =	− ·17	−·26

Equations of Condition.

Position-angle.

Date.	Weight.	sin du.	sin dN.	sin dI.	$2e$ sin Q.	$2e$ cos Q.	s sin dp.	Residual.
1903.		″	″	″	″	″	″	″
Dec. 4	1	−11·2	−5·9	−2·8	+10·9	+ 2·4 =	+0·13	−·11
8	1	−13·9	+1·4	+7·9	− 7·8	+11·4 =	+ ·26	+·11
9	2	−15·4	+4·4	−6·3	+ 6·5	+14·0 =	+ ·25	+·07
10	2	−11·1	−6·4	−1·1	+11·0	+ 0·9 =	+ ·33	+·09
14	1	−14·2	+2·1	+7·9	− 7·4	+12·1 =	+ ·11	−·04
15	1	−14·6	+2·6	−7·2	+ 7·8	+12·4 =	+ ·29	+·08
17	1	−14·5	+2·9	+7·7	+ 6·9	−12·7 =	+ ·05	−·09
30	2	−12·8	−1·8	−7·0	− 9·9	− 8·1 =	+ ·22	−·01
31	2	−11·4	−5·2	+4·6	−10·6	+ 4·2 =	+ ·29	+·07
1904. Jan. 6	1	−11·6	−4·4	+5·7	−10·3	+ 5·4 =	+ ·26	+·05
13	2	−16·9	+7·7	+0·3	+ 0·7	+16·9 =	+ ·23	+·08
14	2	−11·7	−4·6	−4·9	+10·8	+ 4·7 =	+ ·27	+·02
15	3	−12·2	−2·7	+7·2	+ 9·6	− 7·5 =	+ ·26	+·08
19	2	−16·6	+7·0	−2·9	+ 3·6	+16·2 =	+ ·15	−·01
22	1	−16·7	+7·2	−2·3	− 3·1	−16·4 =	+ ·16	·00
Feb. 2	1	−14·0	+2·0	+8·0	+ 7·2	−12·0 =	− ·23	−·37

Date.		Weight.	sin du.	sin dN.	sin dI.	$2e$ sin Q.	$2e$ cos Q.		s sin dp.	Residual.
1904.			″	″	″	″	″		″	″
Feb.	5	4	−14·2	+2·7	+7·9	− 6·8	+12·5	=	+ ·13	−·02
	6	1	−14·6	+2·6	−6·9	+ 8·0	+12·2	=	+ ·07	−·13
	13	2	−10·9	−6·3	+1·8	+10·8	− 1·6	=	+ ·18	−·05
	15	2	−13·4	−0·2	−7·2	− 9·4	− 9·5	=	+ ·30	+·07
	17	2	−15·8	+6·1	+5·3	− 3·5	+15·4	=	+ ·05	−·08
Mar.	10	2	−11·3	−5·1	−3·9	−10·6	− 3·7	=	+ ·13	−·10
	11	4	−12·3	−1·5	+7·7	− 8·8	+ 8·6	=	+ ·19	+·02
	21	2	−14·8	+3·7	−6·2	− 7·1	−13·0	=	+ ·31	+·12
Apr.	6	2	−10·7	−5·7	+3·2	+10·3	− 2·8	=	+ ·26	+·04
	12	1	−10·8	−5·0	+4·6	+10·0	− 4·1	=	+ ·34	+·14
	16	2	−16·0	+7·2	+1·8	− 0·4	+16·0	=	+ ·16	+·03
	18	2	−11·2	−3·8	+6·1	+ 9·5	− 5·8	=	−0·02	−·21

Equations of Condition.

Distance.

Date.		Weight.	sin du.	sin dN.	sin dI.	$2e$ sin Q.	$2e$ cos Q.	$\frac{da}{a}$.		s sin dp.	Residual.
1903.			″	″	″	″	″	″		″	″
Dec.	4	1	+2·4	−3·0	− 0·5	−0·5	−8·6	+16·6	=	+0·10	+·07
	8	1	−5·9	+6·9	− 7·0	+2·2	+8·6	+13·4	=	+ ·12	+·07
	9	2	+4·8	−6·3	− 9·8	+3·4	−6·9	+12·1	=	− ·15	−·21
	10	2	+1·2	−1·4	− 0·1	−0·5	−8·5	+16·8	=	− ·08	−·11
	14	1	−5·8	+6·8	− 7·6	+2·6	+8·4	+13·1	=	+ ·10	+·05
	15	1	+5·6	−7·1	− 8·2	+2·4	−8·1	+12·7	=	+ ·04	−·03
	17	1	−5·7	+6·6	− 8·3	−2·9	−8·1	+12·8	=	− ·21	−·27
	30	2	+5·7	−7·1	− 4·1	−0·3	+9·2	+14·6	=	+ ·10	+·05
	31	2	−3·1	+3·7	− 1·0	+0·2	+8·8	+16·4	=	− ·09	−·12
1904.											
Jan.	6	1	−3·9	+4·7	− 1·7	+0·3	+8·9	+16·0	=	+ ·21	+·17
	13	2	−0·3	−0·4	−12·8	+5·5	0·0	+11·0	=	− ·13	−·20
	14	2	+4·2	−5·2	− 1·6	−0·7	−8·9	+15·8	=	− ·19	−·24
	15	3	−5·0	+6·0	− 3·2	−0·7	−9·1	+15·3	=	+ ·02	−·02
	19	2	+2·3	−3·4	−12·2	+5·0	−3·4	+11·2	=	+ ·18	+·11
	22	1	+1·8	−2·9	−12·4	−5·1	+2·8	+11·1	=	− ·18	−·25
Feb.	2	1	−5·8	+6·6	− 7·4	−2·6	−8·4	+13·2	=	+ ·09	+·03
	5	4	−5·7	+6·4	− 8·0	+3·0	+8·1	+12·9	=	+ ·07	+·01
	6	1	+5·5	−7·2	− 8·1	+2·2	−8·0	+12·5	=	+ ·37	+·30
	13	2	−0·7	+0·8	− 0·1	−0·6	−8·4	+16·7	=	+ ·04	+·01
	15	2	+5·8	−7·5	− 5·6	−0·8	+8·9	+13·6	=	+ ·08	+·02
	17	2	−4·0	+4·1	−11·2	+4·7	+5·1	+11·5	=	+ ·06	−·02

Date.	Weight.	sin du.	sin dN.	sin dI.	$2e$ sin Q.	$2e$ cos Q.	$\frac{da}{a}$.		s sin dp.	Residual.
1903.		$''$	$''$	$''$	$''$	$''$	$''$		$''$	$''$
Mar. 10	2	+3·7	−4·6	− 1·1	+0·9	+8·6	+15·7	=	+ ·06	+·02
11	4	−5·4	+6·2	− 4·2	+1·2	+8·9	+14·4	=	− ·09	−·13
21	2	+4·9	−6·6	− 9·0	−2·8	+7·1	+11·8	=	+ ·24	+·18
Apr. 6	2	−1·7	+2·1	− 0·4	−0·4	−8·2	+16·1	=	+ ·36	+·34
12	1	−2·8	+3·4	− 1·0	−0·4	−8·4	+15·7	=	+ ·26	+·23
16	2	−1·4	+0·9	−12·1	+5·3	+1·5	+10·6	=	+ ·09	+·02
18	2	−4·0	+4·8	− 2·0	−0·5	−8·6	+15·1	=	+ ·22	+·18

Normal Equations.

1902.

	sin du.	sin dN.	sin dI.	$2e$ sin Q.	$2e$ cos Q.	$\frac{da}{a}$.		
sin du	9927	−1504	+ 96	+1131	+1189	+ 930	=	−148·14
sin dN		2684	+ 687	+ 139	− 660	−1602	=	− 4·44
sin dI			3935	+ 141	+ 120	−3295	=	− 31·88
$2e$ sin Q				4241	+1516	− 169	=	− 19·34
$2e$ cos Q					8059	+1260	=	− 15·47
$\frac{da}{a}$						9526	=	+ 53·08

1902–03.

	sin du.	sin dN.	sin dI.	$2e$ sin Q.	$2e$ cos Q.	$\frac{da}{a}$.		
sin du	11430	− 673	− 400	− 406	+1106	+ 222	=	−123·09
sin dN		3060	+ 158	− 280	− 301	− 618	=	− 18·17
sin dI			4120	+ 528	− 69	− 3471	=	− 36·90
$2e$ sin Q				5479	+ 628	− 86	=	+ 7·39
$2e$ cos Q					9307	− 590	=	+ 1·77
$\frac{da}{a}$						13333	=	+ 93·48

1903–4.

	sin du.	sin dN.	sin dI.	$2e$ sin Q.	$2e$ cos Q.	$\frac{da}{a}$.		
sin du	10146	−1547	− 710	− 108	−2758	− 581	=	−127·67
sin dN		2468	− 175	− 334	+1233	+ 496	=	− 5·87
sin dI			4232	− 715	+ 230	− 3579	=	− 8·18
$2e$ sin Q				3956	− 210	+ 563	=	+ 2·70
$2e$ cos Q					8714	− 12	=	+ 26·23
$\frac{da}{a}$						10074	=	+ 34·39

Solutions.

	1902.	1902–3.	1903–4.	Mean.
$\sin du$	$-0{\cdot}01653$	$-0{\cdot}01160$	$-0{\cdot}01458$	$-0{\cdot}0142$
$\sin dN$	$-0{\cdot}00736$	$-0{\cdot}00688$	$-0{\cdot}01250$	$-0{\cdot}0089$
$\sin dI$	$-0{\cdot}00196$	$-0{\cdot}00512$	$-0{\cdot}00349$	$-0{\cdot}0035$
$2e \sin Q$	$+0{\cdot}00051$	$+0{\cdot}00053$	$-0{\cdot}00167$	$-0{\cdot}0002$
$2e \cos Q$	$-0{\cdot}00039$	$+0{\cdot}00163$	$+0{\cdot}00022$	$+0{\cdot}0005$
$\frac{da}{a}$	$+0{\cdot}00533$	$+0{\cdot}00563$	$+0{\cdot}00201$	$+0{\cdot}0043$
du	$-0^{\circ}{\cdot}95$	$-0^{\circ}{\cdot}67$	$-0^{\circ}{\cdot}83$	$-0^{\circ}{\cdot}82$
dN	$-0{\cdot}42$	$-0{\cdot}40$	$-0{\cdot}72$	$-0{\cdot}50$
dI	$-0{\cdot}12$	$-0{\cdot}30$	$-0{\cdot}20$	$-0{\cdot}20$
da	$+0''{\cdot}087$	$+0''{\cdot}092$	$+0''{\cdot}033$	$+0''{\cdot}070$

The Eccentricity of the Orbit.—The three determinations show that the eccentricity is extremely small, the value actually found for $2e \sin Q$ and $2e \cos Q$ being smaller than their probable errors. Mr. H. Struve gave ·01 as the maximum possible value of the eccentricity. The present observations show the limit may be put very much lower. The following values for $2e \sin Q$ and $2e \cos Q$ derived from Dr. Struve's *Memoir* are given for comparison:

Observer.	Epoch.	$2e \sin Q$.	$2e \cos Q$.
Newcomb	1874·5	−·0006	−·0176
Hall	1876·3	−·0068	−·0167
Holden	1876·5	+·0102	+·0002
Hall	1882·1	+·0056	−·0038
Hall	1883·8	+·0109	−·0173
A. Hall (junior)... ...	1892·0	−·0064	−·0168
H. Struve I.	1887·6	−·0100	−·0005
" II.	1889·0	−·0139	−·0056
" III.	1890·6	−·0144	+·0002
" IV.	1892·6	−·0154	−·0062
Greenwich	1902·1	+·0005	−·0004
"	1903·1	+·0005	+·0016
"	1904·1	−·0017	+·0002

The figures suggest that there are in the visual observations some systematic effects due to personal equation from which the photographic observations are free, and that the eccentricity of the orbit is less than 0·001.

Rejecting the terms $2e \sin Q$ and $2e \cos Q$ and combining the three series of normal equations we obtain

	sin du.	sin dN.	sin dI.	$\frac{da}{a}$.	
sin du	31503	−3724	− 1014	+ 571	= −398·90
sin dN		+8212	+ 670	− 1724	= − 28·48
sin dI			+12287	−10345	= − 76·96
$\frac{da}{a}$				+32933	= +180·95

These equations give for the epoch 1903·1 :

$$\sin du = -{\cdot}0139 \pm {\cdot}00081 \quad du = -0^\circ{\cdot}80 \pm {}^\circ{\cdot}08$$
$$\sin dN = -{\cdot}0086 \pm {\cdot}00193 \quad dN = -0^\circ{\cdot}50 \pm {\cdot}11 \quad N = 187^\circ{\cdot}58$$
$$\sin dI = -{\cdot}0034 \pm {\cdot}00128 \quad dI = -0^\circ{\cdot}20 \pm {\cdot}07 \quad I = 117^\circ{\cdot}40$$
$$\frac{da}{a} = +{\cdot}0042 \pm {\cdot}00093 \quad da = +0''{\cdot}069 \pm {}''{\cdot}015 \quad a = 16''{\cdot}202$$

The Longitude and Mean Motion.—Comparison of the value $du = -0^\circ{\cdot}80$ with the values given for different epochs in Dr. Struve's *Memoir* (p. 61) gives a small correction to the longitude for 1890 and to the mean daily motion. The adopted values for 1890·0 $u = 234^\circ{\cdot}42$ and mean daily motion $n = 61^\circ{\cdot}25748$ appear to require corrections of approximately $-0^\circ{\cdot}20$ and $+0^\circ{\cdot}00008$ respectively.

The Mean Distance of the Satellite and Mass of Neptune.—Owing to the great difference in magnitude of *Neptune* and its satellite it is possible that the visual measures are all affected by a small personal equation. The photographic determination is free from this difficulty; and as great care was taken in the determination of the value of the scale, the photographic results would seem specially valuable for this element.

The value found, $a = 16''{\cdot}202$, corresponds to the mean distance of the planet for which $\log \rho = 1{\cdot}47814$. The corresponding value of the mass of *Neptune* is $\frac{1}{M} = 19474$.

The following values were obtained by different observers with the Washington refractor and by Dr. Struve at Pulkowa :

Newcomb	$a = 16''{\cdot}275$	$\frac{1}{M} = 19382$
Holden	16·598	18273
Hall 1875–77	16·482	18662
Hall 1881–82	16·368	19054
Hall 1883–84	16·263	19425
A. Hall (junior)	16·602	18260
H. Struve	16·271	19396
Greenwich...	16·202	19474

The Movement of the Plane of the Satellite's Orbit.—The values $dN = -0^\circ{\cdot}50$, $dI = -0^\circ{\cdot}20$, give for the epoch 1903·1 $N = 187^\circ{\cdot}58$, $I = 117^\circ{\cdot}40$. The following table obtained by taking the simple means of the results of different observations near the same epochs in a fuller table given by Dr. Struve shows the changes in the node and inclination since the discovery of the satellite :

Epoch.	N.	I.	Epoch.	N.	I.
1849·9	179°·12	126°·01	1883·0	184°·36	120°·08
1864·0	181·49	124·20	1890·4	185·27	119·16
1875·8	183·10	121·46	1903·1	187·58	117·40

These changes first pointed out by Marth were explained by Tisserand and Newcomb as arising from the spheroidal shape of *Neptune*. In consequence of the spheroidal figure the orbit will preserve a constant inclination to the equator of *Neptune*, and the node will revolve uniformly on *Neptune's* equator; or in other words the pole of the orbit will describe uniformly a small circle round the pole of the planet. The interesting problem is thus presented of determining the direction of *Neptune's* axis, the inclination of the pole of the orbit to this axis, and the mechanical oblateness of *Neptune* from these changes in the node and inclination.

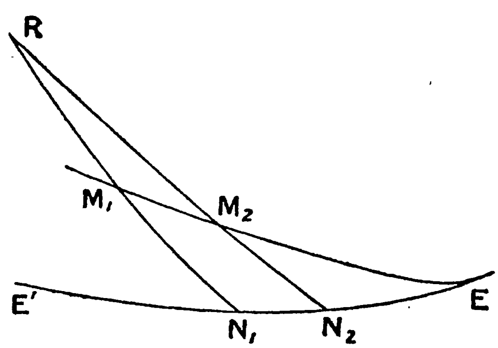

Fig. 1.

In the figure let E′E be the Earth's equator, RM_1N_1 the plane of the satellite's orbit for 1874·0, RM_2N_2 the plane of the orbit for 1896·5, and M_1M_2E the plane of *Neptune's* equator.

Let $\gamma = N_1M_1E$ = the inclination of the orbit of the satellite to *Neptune's* equator;

θ_1 = longitude of this node on *Neptune's* equator at the epoch 1874·0; and

$\psi_1 = M_1N_1$ = the distance between the nodes of the orbit on the two planes of *Neptune's* equator and the Earth's equator.

The differential relations between θ, ψ, γ, and N, I, the node and inclination, are given by the equations

$$\left.\begin{aligned} \sin\gamma d\theta &= -\cos\psi\sin\mathrm{I}d\mathrm{N}+\sin\psi d\mathrm{I} \\ d\gamma &= -\sin\psi\sin\mathrm{I}d\mathrm{N}-\cos\psi d\mathrm{I} \end{aligned}\right\}$$

Putting $$\frac{d\gamma}{dt} = 0 \text{ and } \frac{d\theta}{dt} = \text{const.}$$

we obtain $$\tan\psi = -\frac{d\mathrm{I}}{\sin\mathrm{I}d\mathrm{N}}$$

In this way Dr. Struve finds for the epoch 1874·0, $\psi_1 = 52°·6$ and $\sin\gamma d\theta = 0°·208$, giving the period of revolution of the pole of the orbit round the pole of *Neptune* equal to 1734 sin γ years.

Comparing the present observations with Dr. Struve's in 1890, the changes of N and I in this interval and the value of ψ for the mean date 1896·5 is found :

(H. Struve)	1890·0	N = 185°·15	I = 119°·35
Greenwich	1903·1	N = 187·58	I = 117·40
	1896·5	dN : dI = 2°·43 :	1°·95
		N = 186°·36 .	I = 118·38

Therefore $\psi_2 = 41°·7$ and $\sin\gamma d\theta = 0°·224$ and the time of revolution = 1607 sin γ years.

Comparison of the values of ψ for 1874·0 and 1896·5 may be used to determine a rough value of γ, and of the position of *Neptune's* equator.

We have $N_1M_1 = 52°·6$ $\quad N_2M_2 = 41°·3 \pm 2°·5$

$M_1N_1N_2 = 121·91$ $\quad M_2N_2N_1 = 61·62$

and $N_1N_2 = 3°·40$

Solving the triangles we find

$$EM_2N_2 = \gamma = 22°$$
$$M_2EN_2 = 46°$$
and $$N_2E = 21°$$

Thus the inclination of the orbit to that of *Neptune* is 22°, and the longitude of the node and inclination of *Neptune's* to the Earth's equator are 207° and 134°.

This value of the inclination implies a rotation of the pole of the satellite's orbit in about 600 years. These results are extremely rough, depending as they do on changes in the small quantities dN and dI.

Graphic Solution of the Question.—The positions of the pole of the orbit are given by $\alpha = 90° + \mathrm{N}$ and D = N.P.D. = 180° − I.

For the six epochs considered, the values of α and D are given in the second and third columns of the following table:

Epoch.	R.A. of Pole of Orbit $= \alpha$.	N.P.D. of Pole of Orbit D.	$\sin(\alpha - 270°) \tan\frac{D}{2} = y$.	$\cos(\alpha - 270°) \tan\frac{D}{2} = x$.
1849·9	269°·12	54°·00	−·008	+·510
1864·0	271·49	55·80	+·014	+·529
1875·8	273·10	58·53	+·030	+·559
1883·0	274·36	59·92	+·044	+·574
1890·4	275·27	60·83	+·054	+·585
1903·1	277·58	62·60	+·081	+·603

The best small circle is to be drawn through the six points. In order to exhibit this graphically let a stereographic projection be made by producing lines from the south pole through the six points to meet the tangent plane at the north pole. As a small circle on a sphere is projected into a small circle on the plane, the problem becomes one of plane geometry.

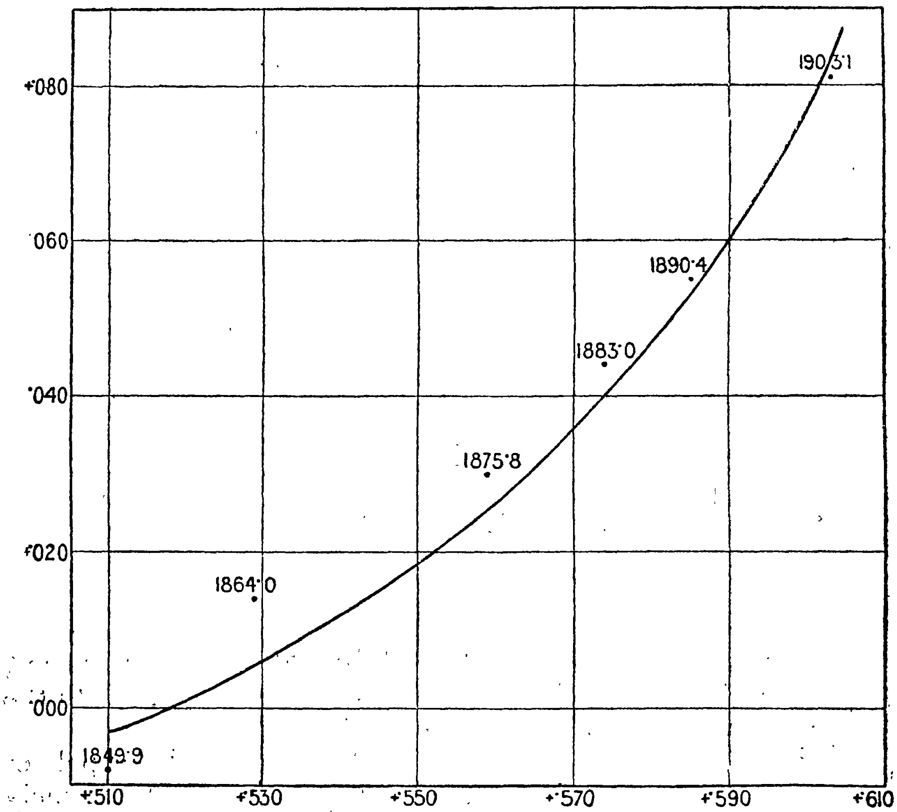

FIG. 2.—Stereographic Projection of Pole of Orbit of *Neptune's* Satellite.

$x = \tan\frac{D}{2}\cos(\alpha - 270°)$ $\quad$ $y = \tan\frac{D}{2}\sin(\alpha - 270°)$

Taking the line whose right ascension is 270° as the initial line, the polar coordinates of the projection of the points α, D are $\tan\frac{D}{2}$ and $\alpha-270°$, and the rectangular coordinates $\tan\frac{D}{2}\cos(\alpha-270°)$ and $\tan\frac{D}{2}\sin(\alpha-270°)$. These are tabulated in the fourth and fifth columns of the above table. Assuming the equation of the circle to be

$$x^2+y^2+fx+gy+c=0$$

the values of f, g, and c are determined by a least-square solution, and the equation of the circle found to be

$$(x-{\cdot}453)^2+(y-{\cdot}152)^2=({\cdot}165)^2$$

This circle is the projection of the small circle whose centre is at R.A. 288° and N.P.D. 50°, and whose radius is 16°, giving 198° and 130° for the longitude of the node and equator of *Neptune*, and 16° for the inclination of the satellite's orbit to *Neptune's* equator.

The result obtained in this way agrees as closely as could be expected with that previously found, and probably closer accordance would have been obtained if more weight had been given to the later and more accurate observations. M. Tisserand, discussing the observations up to 1890, obtained entirely different results, and concluded that the inclination of the orbit of the satellite to the equator of *Neptune* was much greater. The present series of observations and those of Dr. Struve are of so much greater weight than the earlier observations that the value of the material for determining the inclination has been increased considerably. A glance at the diagram on p. 582 shows that observations have not yet been continued long enough for a determination of the inclination to be made with any degree of certainty. It is clearly of great importance that observations of the satellite should be continued, as it would seem that a series of good observations continued over another ten years would fix the position of *Neptune's* axis and the inclination of the orbit to it within a few degrees.

Reduction of the Right Ascensions of the Hong Kong Catalogue of 2120 Southern Stars for the Epoch 1900 to the System of Auwers' Southern Fundamental Stars. By A. M. W. Downing, D.Sc., F.R.S.

From the Preface to the Hong Kong Catalogue of right ascensions of southern stars, recently published, we learn that Auwers' Catalogue, contained in *Astron. Nachrichten,* Nos. 3431–2, was generally used as the standard, but that occasionally